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RECURSION

Recursion is a process in which a problem is defined in terms of itself. In 'C' it is possible to call a function from itself. Functions that call themselves are known as **recursive** functions, i.e. a statement within the body of a function calls the same function. Recursion is often termed as
'Circular Definition'. Thus recursion is the process of defining something in terms of itself. To implement recursion technique in programming, a function should be capable of calling itself.

Example:

```
void main()
{
……………………… /* Some statements*/ fun1();
……………………… /* Some statements */
} void fun1()
{
……………………… /* Some statements */ fun1();

 /*RECURSIVE CALL*/
……………………… /* Some statements */
}
```

Here the function **fun1()** is calling itself inside its own function body, so fun1() is a recursive function. When main() calls fun1(), the code of fun1() will be executed and since there is a call to fun1() insidefun1(), again fun1() will be executed. It looks like the above program will run up to infinite times but generally a terminating condition is written inside the recursive functions which end this recursion. The following program (which is used to print all the numbers starting from the given number to 1 with successive decrement by 1) illustrates this:

```
void main()
{
int a;
printf("Enter a number");
scanf("%d",&a);
fun2(a);
}
int fun2(int b)
{
printf("%d",b);
b--;
if(b>=1) /* Termination condition i.e. b is less than 1*/
{
fun2(b);
}
}
```

How to write a Recursive Function?

Before writing a recursive function for a problem its necessary to define the solution of the problem in terms of a similar type of a smaller problem.

Two main steps in writing recursive function are as follows:

(i). Identify the Non-Recursive part(base case) of the problem and its solution(Part of the problem whose solution can be achieved without recursion).

(ii). Identify the Recursive part(general case) of the problem(Part of the problem where recursive call will be made).

Identification of Non-Recursive part of the problem is mandatory because without it the function will keep on calling itself resulting in infinite recursion.

How control flows in successive recursive calls?

Flow of control in successive recursive calls can be demonstrated in following example:

Consider the following program which uses recursive function to compute the factorial of a number.

```
 void main()
{
int n,f;
printf("Enter a number");
scanf("%d",&n);
f=fact(a);
printf("Factorial of %d is %d",n,f);
}
int fact(int m)
{
int a;
if (m==1)
return (1);
else
a=m*fact(m-1);

return (a);
}
```

In the above program if the value entered by the user is 1 i.e.**n**=1, then the value of **n** is copied into **m**. Since the value of **m** is 1 the condition 'if(m==1)' is satisfied and hence 1 is returned through return statement i.e. factorial of 1 is 1.

When the number entered is 2 i.e. n=2, the value of n is copied into m. Then in function fact(), the condition 'if(m==1)' fails so we encounter the statement a=m*fact(m-1); and here we meet recursion. Since the value of **m** is 2 the expression (m*fact(m-1)) is evaluated to (2*fact(1)) and the result is stored in **a**(factorial of a). Since value returned by fact(1) is 1 so the above expression reduced to (2*1) or simply 2. Thus the expression m*fact(m-1) is evaluated to 2 and stored in **a** and returned to main(). Where it will print 'Factorial of 2 is 2'.

In the above program if **n**=4 then main() will call fact(4) and fact(4) will send back the computed value i.e. **f** to main(). But before sending back to main() fact(4) will call fact(4-1) i.e. fact(3) and wait for a value to be returned by fact(3). Similarly fact(3) will call fact(2) and so on. This series of calls continues until **m** becomes 1 and fact(1) is called, which returns a value which is so called as termination condition.

So we can now say what happened for **n**=4 is as follows

fact(4) returns (4*fact(3))

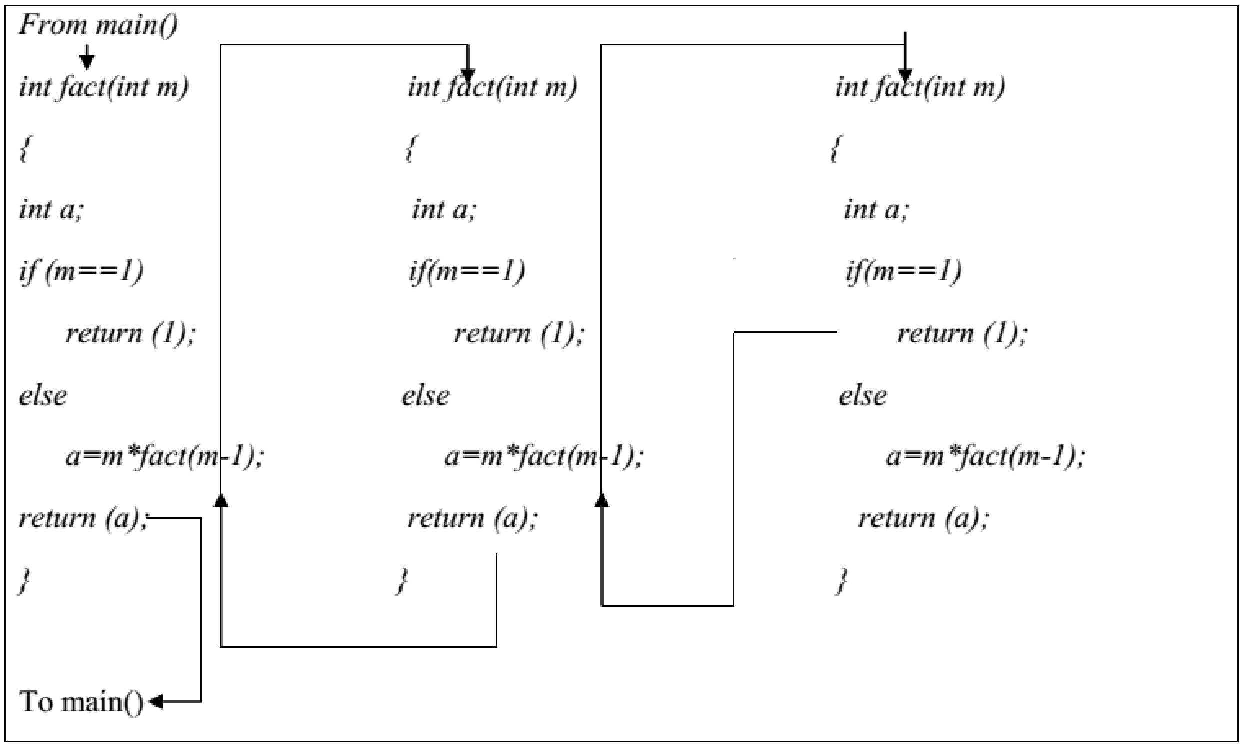
fact(3) returns (3*fact(2))

fact(2) returns (2*fact(1))

fact(1) returns (1)

So for **n**=4, the factorial of 4 is evaluated to 4*3*2*1=24.

For **n**=3, the control flow of the program is as follows:



```
int fact(int m)
{
int a;
if (m==1)
    return (1);
else
    a=m*fact(m-1);
return (a);
}
```

```
int fact(int m)
{
 int a;
if(m==1)
     return (1);
else
     a=m*fact(m-1);
return (a);
}
```

```
int fact(int m)
{
 int a;
 if(m==1)
     return (1);
else
     a=m*fact(m-1);
  return (a);
}
```

Winding and Unwinding phase

All recursive functions work in two phases- winding phase and unwinding phase. Winding phase starts when the recursive function is called for the first time, and ends when the termination condition becomes true in a call i.e. no more recursive call is required. In this phase a function calls itself and no return statements are executed.

After winding phase unwinding phase starts and all the recursive function calls start returning in reverse order till the first instance of function returns. In this phase the control returns through each instance of the function.

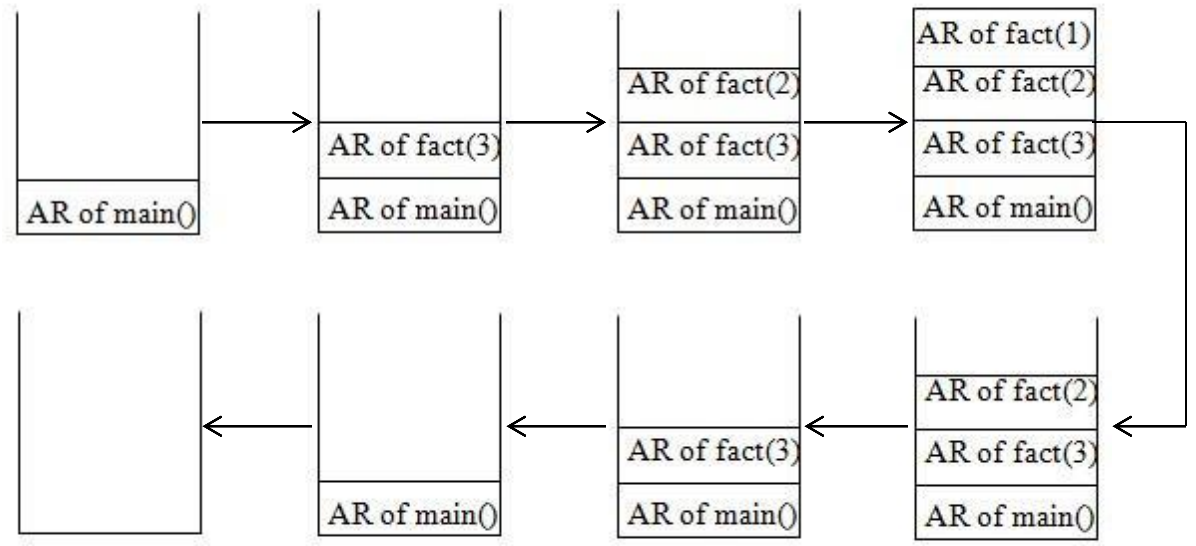
Implementation of Recursion

We came to know that recursive calls execute like normal function calls, so there is no extra technique to implement recursion. All function calls(Whether Recursive or Non-Recursive) are implemented through run time stack. Stack is a Last In First Out(LIFO) data structure. This means that the last item to be stored on the stack(PUSH Operation) is the first one which will be deleted(POP Operation) from the stack. Stack stores Activation Record(AR) of function during run time. Here we will take the example of function fact() in the previous recursive program to find factorial of a number.

Suppose fact() is called from main() with argument 3 i.e.

*fact(3); /*From main()*/*

Now will see how the run time stack changes during the evaluation of factorial of 3.



The following steps will reveal how the above stack contents were expressed:

First main() is called, so PUSH AR of main() into the stack. Then main() calls fact(3) so PUSH AR of fact(3). Now fact(3) calls fact(2) so PUSH AR of fact(2) into the stack. Likewise PUSH AR of fact(1). After the above when fact(1) is completed, POP AR of fact(1), Similarly after completion of a specific function POP its corresponding AR. So when main() is completed POP AR of main(). Now stack is empty.

In the winding phase the stack content increases as new Activation Records(AR) are created and pushed into the stack for each invocation of the function. In the unwinding phase the Activation Records are popped from the stack in LIFO order till the original call returns.

Examples of Recursion

Q1. Write a program using recursion to find the summation of numbers from 1 to n.

Ans: We can say 'sum of numbers from 1 to n can be represented as sum of numbers from 1 to n-1 plus n' i.e.

Sum of numbers from 1 to n = n + Sum of numbers from 1 to n-1
= n + n-1 + Sum of numbers from 1 to n-2
= n+ n-1 + n-2 + ... +1

The program which implements the above logic is as follows:

```
#include<stdio.h>
void main()
{
int n,s;
printf("Enter a number");
scanf("%d",&n);
s=sum(n);
printf("Sum of numbers from 1 to %d is %d",n,s);
}
int sum(int m) int r;
if(m==1)
return (1);
else
r=m+sum(m-1);/*Recursive Call*/
return r;
}
```

Output:

```
Enter a number 5
15
```

Q2. Write a program using recursion to find power of a number i.e. nm.

Ans: We can write,

nm = n*nm-1
=n*n*nm-2
=n*n*n*……………m times *nm-m

The program which implements the above logic is as follows:

```
#include<stdio.h>
int power(int,int);
void main()
{
```

```
int n,m,k;
printf("Enter the value of n and m");
scanf("%d%d",&n,&m);
k=power(n,m);
printf("The value of nm for n=%d and m=%d is %d",n,m,k);
}
int power(int x, int y)
{
if(y==0)
{
return 1;
}
else
{
return(x*power(x,y-1));
}
}
```

Output:

```
Enter the value of n and m
3
5
The value of nm for n=3 and m=5 is 243
```

Q3.Write a program to find GCD (Greatest Common Divisor) of two numbers.

Ans: The GCD or HCF (Highest Common Factor) of two integers is the greatest integer that divides both the integers with remainder equals to zero. This can be illustrated by Euclid's remainder Algorithm which states that GCD of two numbers say x and y i.e.

GCD(x, y) = **x** if y is 0
= **GCD(y, x%y)** otherwise

<u>The program which implements the previous logic is as follows:</u>

```
#include<stdio.h>
int GCD(int,int);
void main()
{
int a,b,gcd;
printf("Enter two numbers"); scanf("%d%d",&a,&b); gcd=GCD(a,b);
printf("GCD of %d and %d is %d",a,b,gcd);
}
int GCD(int x, int y)
{
if(y==0) return x;
else
return GCD(y,x%y);
}
```

Output:

```
Enter two numbers 21
35
GCD of 21 and 35 is 7
```

Q4:Write a program to print Fibonacci Series upto a given number of terms.

Ans: Fibonacci series is a sequence of integers in which the first two integers are 1 and from third integer onwards each integer is the sum of previous two integers of the sequence i.e.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ……..........................

The program which implements the above logic is as follows:

```
#include<stdio.h>
int Fibonacci(int); void main()
{
int term,i;
printf("Enter the number of terms of Fibonacci Series which is going to be
printed");
scanf("%d",&term);
for(i=0;i<term;i++)
{
printf("%d",Fibonacci(i));
}
}
int Fibonacci(int x)
{
if(x==0 || x==1) return 1;
else
return (Fibonacci(x-1) + Fibonacci(x-2));
}
```

Output:

```
Enter the number of terms of Fibonacci Series which is going to be printed 6
1 1 2 3 5 8 13
```

<u>RECURSION VERSES ITERATION</u>

Every repetitive problem can be implemented recursively or iteratively
Recursion should be used when the problem is recursive in nature. Iteration should be used when the problem is not inherently recursive
Recursive solutions incur more execution overhead than their iterative counterparts, but its advantage is that recursive code is very simple.
Recursive version of a problem is slower than iterative version since it requires PUSH and POP operations.
In both recursion and iteration, the same block of code is executed repeatedly, but in recursion repetition occurs when a function calls itself and in iteration repetition occurs when the block of code is finished or a continue statement is encountered.
For complex problems iterative algorithms are difficult to implement but it is easier to solve recursively. Recursion can be removed by using iterative version.

Tail Recursion

A recursive call is said to be tail recursive if the corresponding recursive call is the last statement to be executed inside the function.

Example:

Look at the following recursive function

```
void show(int a)
{
if(a==1)
return;
printf("%d",a);
 show(a-1);
}
```

In the above example since the recursive call is the last statement in the function so the above recursive call is Tail recursive call.
In non void functions(return type of the function is other than void) , if the recursive call appears in return statement and the call is not a part of an expression then the call is said to be Tail recursive, otherwise Non Tail recursive.

Now look at the following example

```
int hcf(int p, int q)
{
if(q==0)
return p;
 else
return(hcf(q,p%q)); /*Tail recursive call*/
}
int factorial(int a)
{
if(a==0)
 return 1;
 else
return(a*factorial(a-1)); /*Not a Tail recursive call*/
}
```

In the above example in hcf() the recursive call is not a part of expression (i.e. the call is the expression in the return statement) in the call so the recursive call is Tail recursive. But in factorial() the recursive call is part of expression in the return statement(a*recursive call) , so the recursive call in factorial() is not a Tail excursive call.

A function is said to be Tail recursive if all the recursive calls in the function are tail recursive.

Tail recursive functions are easy to write using loops,

In tail recursive functions, the last work that a function does is a recursive call, so no operation is left pending after the recursive call returns. Therefore in tail recursive functions , there is nothing to be done in unwinding phase, so we can jump directly from the last recursive call to the place where recursive function was first called.

Tail recursion can be efficiently implemented by compilers so we always will try to make our recursive functions tail recursive whenever possible.

Functions which are not tail recursive are called augmentive recursive functions and these types of functions have to finish the pending work after the recursive call finishes.

Indirect and Direct Recursion

If a function fun1() calls another function fun2() and the function fun2() in turn calls function fun1(), then this type of recursion is said to be **indirect** recursion, because the function fun1() is calling itself indirectly.

```
fun1( )
{
……………………… /* Some statements*/ fun2();
……………………… /* Some statements*/
}
fun2( )
{
……………………… /* Some statements*/ fun1();
```

```
                    ……………………… /* Some statements*/
                    }
```

The chain of functions in indirect recursion may involve any number of functions.For example suppose n number of functions are present starting from f1() to fn() and they are involved as following: f1() calls f2(), f2() calls f3(), f3() calls f4() and so on with fn() calls f1().

If a function calls itself directly i.e. function fun1() is called inside its own function body, then that recursion is called as direct recursion. For example look at the following:

```
                    fun1()
                    {
                    ... /* Some statements*/
                    fun2();
                    ... /* Some statements*/
                              }
```

Indirect recursion is complex, so it is rarely used.

Exercise:
Find the output of programs from 1 to 5.

```
1. void main()
          {
          printf("%d\n",count(17243));
          }
          int count(int x)
          {
          if(x==0)
          return 0;
          else
          return 1+count(x/10)
          }

2. void main()
          {
          printf("%d\n",fun(4,8));
          printf("%d\n",fun(3,8));
          }
          int fun(int x. int y)
          {
          if(x==y)
```

```
        return x;
        else
        return (x+y+fun(x+1,y-1));
        }

3. void main()
        {
        printf(“%d\n”,fun(4,9));
        printf(“%d\n”,fun(4,0));
        printf(“%d\n”,fun(0,4));
        }
        int fun(int x, int y)
        {
        if(y==0)
        return 0;
        if(y==1)
        return x;
        return x+fun(x,y-1);
        }

4. void main()
        {
        printf(“%d\n”,fun1(14837));
        }
        int fun1(int m)
        {
        return ((m)? m%10+fun1(m/10):0);
        }

5. void main()
        {
         printf(“%d\n”,fun(3,8)); }
        int fun(int x, int y)
        {
        if(x>y)
        return 1000;
        return x+fun(x+1,y);
        }
```